FREE CHOICE AND HOMOGENEITY

1 Introduction

In the last few decades there has been a flurry of research into the apparent validity of the 'Free Choice' inference, where sentences like (1) and (3) appear to imply sentences like (2) and (4).[1]

(1) You may have soup or salad.

(2) You may have soup and you may have salad.

(3) Mary might be in New York or Los Angeles.

(4) Mary might be in New York and Mary might be in Los Angeles.

FREE CHOICE $\Diamond(\mathsf{A} \vee \mathsf{B}) \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$

Free Choice is a surprising inference from the perspective of a classical possible worlds semantics for modals, and a boolean semantics for disjunction. On that theory, (1) simply says that there is some accessible world where either you have soup or you have salad. This does not imply that there is both an accessible world where you have soup and an accessible world where you have salad. For this reason, semantic analyses of Free Choice have all offered some kind of revision to this classical semantics.[2]

Free Choice also challenges a classical logic for possibility modals and disjunction. Kamp 1974 showed that there is serious tension between the validity of Free Choice, Disjunction Introduction, and the Upwards Monotonicity of possibility modals:

DISJUNCTION INTRODUCTION $\mathsf{A} \models \mathsf{A} \vee \mathsf{B}$

UPWARDS MONOTONICITY If $\mathsf{A} \models \mathsf{B}$, then $\Diamond\mathsf{A} \models \Diamond\mathsf{B}$

For suppose that the entailment relation is transitive:

TRANSITIVITY If $\mathsf{A} \models \mathsf{B}$ and $\mathsf{B} \models \mathsf{C}$, then $\mathsf{A} \models \mathsf{C}$

With these assumptions, Free Choice implies the equivalence of any two possibility claims:

EXPLOSION $\Diamond\mathsf{A} \models \Diamond\mathsf{B}$

Fact 1 (Kamp)**.** Transitivity, Disjunction Introduction, Upwards Monotonicity and Free Choice imply Explosion.

[1] See von Wright 1968 and Kamp 1974, Kamp 1978.

[2] For semantic accounts of free choice, see among others Asher and Bonevac 2005, Aher 2012, Aloni 2007, Barker 2010, Ciardelli et al. 2009, Charlow 2015, Fusco 2015, Geurts 2005, Roelofsen 2013, Simons 2005, Starr 2016, Willer 2017a, and Zimmermann 2000.

After all, $\Diamond A$ implies $\Diamond(A \vee B)$ by Disjunction Introduction and Upwards Monotonicity, which by Free Choice implies $\Diamond B$. So by Transitivity $\Diamond A$ implies $\Diamond B$. For this reason, all existing accounts of Free Choice have rejected the inference from $\Diamond A$ to $\Diamond(A \vee B)$, by giving up either Disjunction Introduction or Upwards Monotonicity.

A semantic account of Free Choice requires major revision to a classical logic and semantics for disjunction and modals. Any such account also faces serious problems stemming from negation. Alonso-Ovalle 2006 presented what may be the largest challenge for semantic accounts of Free Choice. Like scalar implicatures, Free Choice seems to disappear under negation. While (1) implies (2), the negation of (2) does not appear to imply the negation of (1). Quite the opposite: (5) actually appears to imply (6):

(5) You can't have soup or salad.

(6) You can't have soup and you can't have salad.

DUAL PROHIBITION $\neg\Diamond(A \vee B) \models \neg\Diamond A \wedge \neg\Diamond B$

While the classical semantics above does not validate Free Choice, it does validate Dual Prohibition. If there is no accessible world where you have soup or salad, there cannot be an accessible world where you have soup, or one where you have salad, since you would have soup or salad in either such world.

There are several reasons to think that Free Choice and Dual Prohibition are incompatible. First, suppose that we accept the rules of Transitivity and Contraposition.

CONTRAPOSITION If $A \models B$, then $\neg B \models \neg A$.

Then Free Choice and Dual Prohibition again imply that any two possibility claims are equivalent.

Fact 2. Transitivity, Contraposition, Free Choice, and Dual Prohibition imply Explosion.

Proof. By Free Choice, $\Diamond(A \vee B) \models \Diamond B$. So by Contraposition $\neg\Diamond B \models \neg\Diamond(A \vee B)$. By Dual Prohibition, $\neg\Diamond(A \vee B) \models \neg\Diamond A$. So by Transitivity, $\neg\Diamond B \models \neg\Diamond A$ and hence by Contraposition again $\Diamond A \models \Diamond B$. □

For this reason, the few defenders of both Free Choice and Dual Prohibition (Starr 2016, Willer 2017a) have given up the rule of Contraposition.

But even giving up Contraposition isn't enough to avoid further trouble. The joint acceptance of Free Choice and Dual Prohibition also requires further nonclassicality in the logic of disjunction. Consider the Law of Excluded Middle and Constructive Dilemma:

LEM $\models A \vee \neg A$

CONSTRUCTIVE DILEMMA If $A \models C$ and $B \models D$, then $A \vee B \models C \vee D$

These two principles can be accepted independently of Disjunction Introduction. But they also appear incompatible with the validity of Free Choice and Dual Prohibition.

For suppose we again accept that entailment is transitive. Then Free Choice, Dual Prohibition and the above assumptions lead to an absurd result, which we can call the 'Interconnectedness of All Things':

IAT $\models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$

IAT is bizarre, because it seems inconsistent with the intuitive idea that there can be two claims that differ with respect to their modal status. Unfortunately, it follows from the assumptions above:

Fact 3. Transitivity, Free Choice, Dual Prohibition, Constructive Dilemma and LEM imply IAT.

Proof. By LEM, $\models \Diamond(A \vee B) \vee \neg\Diamond(A \vee B)$. By Free Choice, Dual Prohibition, and Constructive Dilemma, $\Diamond(A \vee B) \vee \neg\Diamond(A \vee B) \models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$. So by Transitivity, $\models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$. □

To see the problem informally, suppose IAT is false, so that there is a situation in which A is possible and B is not. It then seems that neither $\Diamond(A \vee B)$ nor $\neg\Diamond(A \vee B)$ can hold, since the former requires that both A and B are possible, while the latter requires that both are impossible.

Summing up, Free Choice looks like an intuitively plausible principle. But validating Free Choice semantically comes with a variety of costs. Previous attempts to validate Free Choice have given up a variety of classical principles, including Disjunction Introduction, Upwards Monotonicity, Contraposition, Dual Prohibition, LEM, and Constructive Dilemma.

In the face of these concerns, there are three natural strategies to pursue. Some, like Alonso-Ovalle 2006, have accepted Dual Prohibition but rejected the semantic validity of Free Choice, instead offering a pragmatic account of its validity.[3] Others, like Barker 2010, have offered the reverse diagnosis, validating Free Choice while offering a pragmatic account of Dual Prohibition. Any such attempt, though, will still have to give up some of the classical assumptions in Fact 1. Finally, a few recent papers (Starr 2016; Willer 2017a) have offered complex new semantics for negation, possibility, and disjunction that validate both Free Choice and Dual Prohibition simultaneously. Any such semantics seems like it will have to be complex, since it requires some response to each of the three incompatibility results above.

In this paper, I develop a new semantics for Free Choice that validates Dual Prohibition while avoiding all of the results above in a unified way.[4] Every one of the classical principles above remains valid, except for one: the transitivity of entailment. To achieve this solution, the main theoretical insight is to introduce homogeneity effects (von Fintel 1997; Kriz 2015) into the semantics of Free Choice. The key idea will be that disjunctive possibility claims are defined only when either both or neither of A and B are possible.

(7) Mary may have soup or salad.

[3] For pragmatic accounts of Free Choice, see among others: Alonso-Ovalle 2006; Fox 2007; Franke 2011; Klinedinst 2007; Kratzer and Shimoyama 2002; Romoli and Santorio 2017; and Schulz 2005.

[4] In this paper, I won't argue directly against the various pragmatic accounts of Free Choice. One place to look for concerns about various pragmatic analyses is a growing body of literature suggesting that Free Choice differs from scalar implicature with respect to processing time (Chemla and Bott 2014) and acquisition (Tieu et al. 2016). But rather than address pragmatic accounts directly, I will simply develop a semantics that validates both Free Choice and Dual Prohibition. This undercuts one of the main arguments against semantic accounts of Free Choice, but a further comparison between semantic and pragmatic accounts is still needed.

TRUE iff Mary may have soup and Mary may have salad.
FALSE iff Mary can't have soup and Mary can't have salad.
UNDEFINED otherwise.

By relying on homogeneity in the right way, we can straightforwardly validate both Free Choice and Dual Prohibition. Free Choice is valid because whenever $\Diamond(\mathsf{A} \vee \mathsf{B})$ is true, either both or neither of A and B must be possible. If neither were possible, then $\Diamond(\mathsf{A} \vee \mathsf{B})$ would be false; so both must be true. Dual Prohibition is valid for the same reason. In order for $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$ to be true, either both or neither of A and B must be possible. If both were possible, $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$ would be false; so both must be impossible.

Once we incorporate homogeneity effects into our semantics, we can also introduce a definition of entailment that is sensitive to definedness: Strawson entailment. Here, an argument is valid only if it is truth preserving in contexts where the conclusion is defined. Crucially, this notion of entailment will allow us to preserve all of the classical principles above, except the transitivity of entailment. To foreshadow, $\Diamond\mathsf{A}$ will imply $\Diamond(\mathsf{A} \vee \mathsf{B})$, because to evaluate this inference we hold fixed the homogeneity of the possibility of A and B. For the same reason, $\Diamond(\mathsf{A} \vee \mathsf{B})$ will imply $\Diamond\mathsf{B}$. But these inferences together will not require that $\Diamond\mathsf{A}$ implies $\Diamond\mathsf{B}$, because this last inference has no homogeneity effect. The failure of Transitivity responds in a similar way to the incompatibility results regarding Dual Prohibition.

The last main feature of the proposal below is its generality. We will see that homogeneity can be incorporated into very different semantic frameworks, with quite similar results. In particular, in the rest of this paper we develop two new semantics: homogenous alternative semantics, and homogenous dynamic semantics. Each theory tells a different story about the compositional origins of homogeneity. In homogenous alternative semantics, the effect is contributed by the semantics of possibility modals. In homogenous dynamic semantics, the effect is contributed by the meaning of disjunction. The upshot of all this is that homogeneity effects offer an attractive tool for a wide variety of defenders of the semantic validity of Free Choice: many such theories can go on to validate Dual Prohibition and preserve much of classical logic by using a single straightforward idea.

2 Alternative semantics

In the first half of this paper, we will validate Free Choice and Dual Prohibition with a new theory, *homogenous alternative semantics*, that incorporates homogeneity effects into alternative semantics. Let's start by reviewing how alternative semantics can be used to explain Free Choice, as in Simons 2005 and Aloni 2007. We will then enrich alternative semantics with homogeneity effects.

In alternative semantics, sentence meanings are not propositions, but instead sets of propositions (or 'alternatives'). The idea is that a disjunction $\mathsf{A} \vee \mathsf{B}$ presents both of A and B as alternatives.

Definition 1. $[\![\mathsf{A} \vee \mathsf{B}]\!] = \{[\![\mathsf{A}]\!], [\![\mathsf{B}]\!]\}$

Disjunction contributes a set of propositions as its meaning. To validate Free Choice, we let possibility modals operate on each alternative in this set.

We can theorize about alternative semantics with some generality by deriving the meaning of possibility modals ($\Diamonddot$) from an underlying operator $\Diamond$—the 'proto-possibility' operator—which maps a proposition to a new proposition. For example, we might assume that $\Diamond$ is a usual Kripkean modal operator, which existentially quantifies over accessible worlds (Kripke 1963; Kratzer 2012):

Definition 2. $\llbracket \Diamond \mathsf{A} \rrbracket = \{w \mid \exists v : wRv \;\&\; v \in \llbracket \mathsf{A} \rrbracket\}$

The proto-possibility operator regulates the behavior of the possibility modal $\Diamonddot$ when the prejacent is not an alternative. It also helps determine how $\Diamonddot$ behaves when its antecedent denotes a non-trivial sets of alternatives. We illustrate this for the case in which $\llbracket \mathsf{A} \rrbracket$ denotes a set of propositions.

Definition 3. $\llbracket \Diamonddot \mathsf{A} \rrbracket = \bigcap \{\llbracket \Diamond \rrbracket(\mathbf{A}) \mid \mathbf{A} \in \llbracket \mathsf{A} \rrbracket\}$

To simplify a bit more, suppose the set of propositions in $\llbracket \mathsf{A} \rrbracket$ is $\{\mathbf{A}_1, ..., \mathbf{A}_n\}$, denoted by the sentences $\mathsf{A}_1, ..., \mathsf{A}_n$. Then $\Diamonddot \mathsf{A}$ is true just in case each of the possibility claims $\Diamond \mathsf{A}_1, \dots, \Diamond \mathsf{A}_n$ is true. In other words, the alternative sensitive possibility modal is a generalized conjunction of a series of proto-possibility claims, distributed over the prejacent's alternatives. To recycle one of our early examples, the truth-conditions of *Mary may have soup or salad* demands the truth of both: *Mary may have soup* and *Mary may have salad*.

Before showing how this framework engages with the incompatibility results from §1, we must make some bookkeeping adjustments. Once we access the higher type of sets of propositions, we need a route connecting them back with propositional meanings. Without such a route, we would not be able to make sense of logical consequence. Furthermore, and relatedly, Definition 3 does not provide for non-disjunctive prejacents without such a bridge.

These problems are often solved by introducing a closure operator ! and type raiser $\uparrow$ to move from propositions to sets of propositions, and back (Kratzer and Shimoyama 2002). ! maps a set of propositions to its union, while $\uparrow$ takes a proposition to its singleton. However, introducing such operations into our semantics would considerably complicate our LFs and make it difficult to consider our incompatibility results directly. This is because we would have to reinterpret what exactly Free Choice, Dual Probibition, and other principles even say. For example it would be difficult to say whether Free Choice involves a violation of Upwards Monotonicity since Disjunction Introduction might be analyzed as $\mathsf{A} \models !(\mathsf{A} \vee \mathsf{B})$ rather than $\mathsf{A} \models \mathsf{A} \vee \mathsf{B}$.

Instead, we opt for a different approach. First, we avoid the need for a type raising operation by defining our possibility modal polymorphically. $\Diamonddot$ can either take a proposition or a set of propositions as input. When it takes a proposition as input, it simply applies $\Diamond$; otherwise, it universally quantifies over alternatives.

Definition 4. $\llbracket \Diamonddot \mathsf{A} \rrbracket = \begin{cases} \llbracket \Diamond \mathsf{A} \rrbracket & \text{if } \llbracket \mathsf{A} \rrbracket \subseteq W \\ \bigcap \{\llbracket \Diamond \rrbracket(\mathbf{A}) \mid \mathbf{A} \in \llbracket \mathsf{A} \rrbracket\} & \text{otherwise.} \end{cases}$

Instead of an explicit type raising operator, we invoke an explicit existential closure operator !. And instead of obligatorily placing this operator in LF, we use it only to define entailment. Just like our possibility modal, we can define our closure operator polymorphically. When $\llbracket \mathsf{A} \rrbracket$ is a proposition, ! has no effect on A. But when $\llbracket \mathsf{A} \rrbracket$ is a set of propositions, ! takes the union of all of the A alternatives.

Definition 5. $[\![!\mathsf{A}]\!] = \begin{cases} [\![\mathsf{A}]\!] & \text{if } [\![\mathsf{A}]\!] \subseteq W \\ \bigcup [\![\mathsf{A}]\!] & \text{otherwise.} \end{cases}$

Say that $!\mathsf{A}$is the closure of A. Then an argument is valid just in case the closure of the conclusion is true whenever the closure of all the premises are true.

Definition 6. $\mathsf{A}_1, \ldots, \mathsf{A}_n \models \mathsf{C}$ iff $\bigcap_{i \in [1,n]} [\![!\mathsf{A}_i]\!] \subseteq [\![!\mathsf{C}]\!]$

This proposal guarantees that disjunction behaves as classically as possible. Since entailment is only sensitive to the closed form of a sentence, the alternative sensitive disjunction *or* must satisfy Disjunction Introduction, LEM, and Constructive Dilemma. In addition, the proposal yields the DeMorgan equivalence of $\mathsf{A} \vee \mathsf{B}$ and $\neg(\neg\mathsf{A} \wedge \neg\mathsf{B})$.

A further consequence of the above is that logical equivalence is less fine grained than equivalence of meaning. While $\mathsf{A} \vee \mathsf{B}$ and $\neg(\neg\mathsf{A} \wedge \neg\mathsf{B})$ are co-entailing, they do not have the same meaning. The disjunction, but not the negated conjunction, denotes a set of alternatives. For this reason, our logic for possibility modals is hyperintensional in the sense that substituting logical equivalents in modal prejacents does not guarantee equivalence of the resulting possibility claims. Possibility modals with disjunctive prejacents go in for Free Choice, while modals with negated conjunctions in the prejacent do not.[5]

Now let's turn to the principles in §1. To consider Free Choice, note that we have the following identity, regardless of what $\Diamond$ means:

$$[\![\Diamond(\mathsf{A} \vee \mathsf{B})]\!] = [\![\Diamond\mathsf{A}]\!] \cap [\![\Diamond\mathsf{B}]\!]$$

This evidently guarantees that Free Choice is valid, holding fixed a boolean semantics for conjunction. In addition, we already saw that Disjunction Introduction is valid. Furthermore, since the consequence relation simply involves preservation of truth, it is transitive. So to avoid the problems from Fact 1, this semantics gives up Upwards Monotonicity. In particular, although A implies $\mathsf{A} \vee \mathsf{B}$, $\Diamond\mathsf{A}$ does not imply $\Diamond(\mathsf{A} \vee \mathsf{B})$. The reason for this is that the inference from A to $\mathsf{A} \vee \mathsf{B}$ treats the disjunction $\mathsf{A} \vee \mathsf{B}$ as closed, while the inference from $\Diamond\mathsf{A}$ to $\Diamond(\mathsf{A} \vee \mathsf{B})$ does not.

While Free Choice is valid in this semantics, Dual Prohibition is not. For suppose that we hold fixed a boolean semantics for negation. Now suppose that there is a scenario in which $\Diamond\mathsf{A}$ is true and $\Diamond\mathsf{B}$ is false. In this case we have $\neg\Diamond(\mathsf{A} \vee \mathsf{B})$, and so Dual Prohibition fails. More generally, the validity of Dual Prohibition corresponds to the trivializing condition that any possibility claim implies any other. Summarizing:

Observation 1.

1. For any operator $\Diamond$, $\Diamond(\mathsf{A} \vee \mathsf{B}) \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$.
2. For any operator $\Diamond$, if $\Diamond\mathsf{A}; \neg\Diamond\mathsf{B}$ is consistent, then $\Diamond$ does not validate Dual Prohibition.

[5] Although see Willer 2017a for arguments that Free Choice occurs even in this case, and Ciardelli et al. 2018 for a response.

One last remark: the alternatives approach does not require that possibility modals always go in for Free Choice. It is compatible with what we have said to have the closure operator ! occur in the prejacent, generating the form $\Diamond!(\mathsf{A} \vee \mathsf{B})$. This would account for some localized failures of Free Choice, such as (8) and (9).[6]

(8) Mary may have soup or salad, but I don't know which.

(9) Mary may have soup or salad, but I won't tell you which.

3 Homogeneity

In §2, we reviewed how an alternative semantics for disjunction can be exploited to validate Free Choice. We saw that in addition to giving up the Upwards Monotonicity of possibility modals, the semantics also invalidated Dual Prohibition. This latter result in particular is a significant problem for the view. Now we will introduce the main tool that we will exploit in what follows: homogeneity effects.

Homogeneity effects have been used to explain apparent violations of excluded middle for both conditionals and plurals.[7] Here is the problem: observe first that predications involving plural definites, like (10), plausibly license inferences to universal claims like (11).

(10) The cherries in my yard are ripe.

(11) All the cherries in my yard are ripe.

If some but not all cherries are ripe, one would not be in a position to assert (10). Furthermore, plural definites plausibly accept the law of excluded middle.[8] That is, the following sounds like a logical truth:

(12) Either the cherries in my yard are ripe or they (=the cherries in my yard) are not ripe.

If someone were to utter (12), they would sound just about as informative as if they had made a tautological statement (although you might learn from it that they have cherries in their yard). The problem is that, starting with (12) and exploiting entailments like the one from (10) to (11) as well as Constructive Dilemma, we can reason our way to (13):

(13) Either all the cherries in my yard are ripe or all the cherries in my yard are not ripe.

That seems puzzling: did we just prove from logical truths and valid inferences that my yard cannot have some ripe cherries and some non-ripe ones? Of course, something must have gone wrong. The homogeneity view of plural definites explains what that is:

[6] See Zimmermann 2000 among others for discussion.

[7] See for example von Fintel 1997 and Kriz 2015 for discussion. These accounts differ in the exact status of homogeneity. In von Fintel 1997, homogeneity is treated as a kind of presupposition, while in Kriz 2015 it is a different kind of undefinedness. For our results, we will not need to take a stand on this issue; we will, however, need to follow von Fintel 1997 in relying on Strawson entailment as our central notion of validity.

[8] Here we differ from Kriz 2015.

first, plural definites are defined only when either the *F*'s are either homogenously *G*'s or homogenously not *G*'s. If this condition is satisfied, their content is that all *F*'s are *G*'s. The sense in which (12) sounds tautological is that it cannot be false if its definedness condition is satisfied. Similarly, the sense in which (10) entails (11) is that if the definedness conditions of (10) is satisfied and (10) is true, (11) cannot fail to be true. But even if we can exploit these to deduce (13) we do not have license us to claim that (13) is valid: our justification for (12) and for the (10)-(11) entailment did not discharge the homogeneity condition.

von Fintel 1997 proposes a similar analysis of conditionals. The idea is that the conditional $\mathsf{A} \to \mathsf{C}$ says that all of the closest A worlds are C worlds. But in addition, the conditional carries a definedness condition that either all of those worlds make C true, or they all make C false. This explains the felt equivalence of (14) and (15), for example.[9]

(14) Mary doubts that if Alex is at the party, Billy is.

(15) Mary thinks that if Alex is at the party, Billy isn't.

In what follows, we will see how to incorporate this basic idea in the different setting of possibility modals.

4 Homogenous alternative semantics

We will now develop a new semantics for Free Choice by integrating homogeneity effects into alternative semantics. The key idea is that $\Diamond(\mathsf{A} \vee \mathsf{B})$ is defined only when either all of the alternatives to $\mathsf{A} \vee \mathsf{B}$ are possible, or all of them are impossible. The alternatives must be homogeneous with respect to modal status.

In homogenous alternative semantics, we can implement this idea by letting possibility modals themselves contribute definedness conditions.[10] (Later, with homogenous dynamic semantics, we will explore whether these definedness conditions could instead be contributed by disjunction.) So we will say that $\Diamond\mathsf{A}$ is defined only if either all alternatives in $[\![\mathsf{A}]\!]$ are possible, or all are impossible. Furthermore, since $\Diamond$ incorporates alternative sensitivity into its definedness conditions, we can let its truth conditions be classical, operating on the closed form of its prejacent. As before, we will define $\Diamond$ polymorphically, so that it behaves classically whenever its prejacent is propositional.

Definition 7.

1. If $[\![\mathsf{A}]\!] \in \langle s, t\rangle$, then $[\![\Diamond\mathsf{A}]\!] = [\![\Diamond\mathsf{A}]\!]$.
2. Otherwise $[\![\Diamond\mathsf{A}]\!](w)$ is defined only if either:
   (a) $\bigcap\{[\![\Diamond]\!](\mathbf{A}) \mid \mathbf{A} \in [\![\mathsf{A}]\!]\}(w) = 1$ or
   (b) $\bigcup\{[\![\Diamond]\!](\mathbf{A}) \mid \mathbf{A} \in [\![\mathsf{A}]\!]\}(w) = 0$

   If defined, $[\![\Diamond\mathsf{A}]\!](w) = [\![\Diamond !\mathsf{A}]\!](w)$.

[9] See Cariani and Santorio Forthcoming for recent discussion.

[10] For this reason, we will now treat meanings as partial functions from worlds to truth values rather than simply as sets of worlds.

To make predictions about the results in §1, we also need to make the appropriate assumptions about negation and conjunction. These connectives must allow homogeneity effects to project in the right way. So we will assume that negations carry the same definedness conditions as their inputs, and that a conjunction is defined only when each conjunct is.[11] [12]

Definition 8.

1. $[\![\neg \mathsf{A}]\!](w)$ is defined only if $[\![\mathsf{A}]\!](w)$ is defined.
   If defined, $[\![\neg \mathsf{A}]\!](w) = 1 - [\![\mathsf{A}]\!](w)$.

2. $[\![\mathsf{A} \wedge \mathsf{B}]\!](w)$ is defined only if $[\![\mathsf{A}]\!](w)$ and $[\![\mathsf{B}]\!](w)$ are defined.
   If defined, $[\![\mathsf{A} \cap \mathsf{B}]\!](w) = min([\![\mathsf{A}]\!](w), [\![\mathsf{B}]\!](w))$.

In addition, we also need to enrich disjunction and our closure operator with projection properties. We will assume that a disjunction is defined only if both disjuncts are defined. When defined, it will form the set of disjunct meanings. In addition, we will assume that the closure of $\mathsf{A} \vee \mathsf{B}$ is defined only if A and B are each defined.

Definition 9. $[\![\mathsf{A} \vee \mathsf{B}]\!](w)$ is defined only if $[\![\mathsf{A}]\!](w)$ and $[\![\mathsf{B}]\!](w)$ are defined.
If defined, $[\![\mathsf{A} \vee \mathsf{B}]\!](w) = \{[\![\mathsf{A}]\!], [\![\mathsf{B}]\!]\}$

Definition 10.

1. If $[\![\mathsf{A}]\!] \in \langle s, t \rangle$, then $[\![!\mathsf{A}]\!] = [\![\mathsf{A}]\!]$.

2. Otherwise $[\![!\mathsf{A}]\!](w)$ is defined only if $\mathbf{A}(w)$ is defined for every $\mathbf{A} \in [\![\mathsf{A}]\!]$.
   If defined, $[\![!\mathsf{A}]\!] = \lambda w.\exists \mathbf{A} \in [\![\mathsf{A}]\!] : \mathbf{A}(w) = 1$

Finally, to get our desired predictions, we need a definition of consequence. One leading candidate for languages involving definedness conditions is Strawson-validity (Strawson, 1952; von Fintel, 1997, 1999, 2001). According to this notion, an argument is valid just in case the conclusion is true whenever the conclusion is defined and the premises are true. As in §2, we also assume that entailment is sensitive to the closed forms of sentences.

Definition 11. Some premises $\mathsf{A}_1, \ldots \mathsf{A}_n$ entail C just in case $[\![!\mathsf{C}]\!](w) = 1$ whenever:

1. $[\![!\mathsf{A}_1]\!](w) = 1; \ldots; [\![!\mathsf{A}_n]\!](w) = 1$.

2. $[\![!\mathsf{C}]\!](w)$ is defined.

We now have all of the tools necessary in order to solve the problems from §1. In the next section we turn to this task.

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[11] For simplicity, we assume that negation and conjunction must take propositions rather than sets of alternatives as meanings. So in order for disjunction to embed under these expressions, our closure operator would have to be applied.

[12] Here, we assume that a conjunction is defined only if each conjunct is defined. This assumption is slightly stronger than what we need in order to get our desired predictions. For example, we could also allow a more complex pattern of projection for conjunction, where the second conjunct treats the first conjunct as part of its local context, as in Heim 1992. But this won't be relevant in what follows, so we stick to the current formulation for simplicity.

5 Results

With all of our assumptions in place, let's turn to the principles from §1. First, the semantics validates Free Choice.

Observation 2. $\Diamond(\mathsf{A} \vee \mathsf{B}) \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$

Whenever $\Diamond(\mathsf{A} \vee \mathsf{B})$ is true, it is defined. If the sentence is defined, either $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$ are both true, or they are both false. But the truth conditions for $\Diamond(\mathsf{A} \vee \mathsf{B})$ require that at least one of $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$ is true; so together this all requires that both are true.

Observation 2 follows from a more general property. Any inference that was valid in the alternative semantics from §2 remains valid when enriched with homogeneity effects. After all, homogenous alternative semantics makes possibility sentences true in the same worlds as in §2, provided that the sentence is defined. Furthermore, it takes less for an inference to be valid in homogenous alternative semantics, since we can restrict our attention to the worlds where the conclusion is defined.

Since homogenous alternative semantics validates Free Choice, it must respond to Fact 1, which concerned the classical properties of disjunction. As in ordinary alternative semantics, Disjunction Introduction remains valid, and for the same reason as before.

Observation 3. $\mathsf{A} \models \mathsf{A} \vee \mathsf{B}$

Again, when we consider whether A implies $\mathsf{A} \vee \mathsf{B}$, we restrict attention to the closed form of $\mathsf{A} \vee \mathsf{B}$, the worlds where one disjunct is true.

At this point, we have our first departure from traditional alternative semantics. Unlike the theory in §2, homogenous alternative semantics validates Upwards Monotonicity. In particular, since Disjunction Introduction is valid, this theory validates the inference from $\Diamond\mathsf{A}$ to $\Diamond(\mathsf{A} \vee \mathsf{B})$, which some have thought is directly incompatible with Free Choice.[13]

Observation 4. $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$

The definedness of the conclusion requires that either both or neither of A and B are possible. Combined with the truth of the premise $\Diamond\mathsf{A}$, this means that $\Diamond\mathsf{B}$ is true, and so $\Diamond(\mathsf{A} \vee \mathsf{B})$ is true.

One might worry that this last result is too strong, because it fails to distinguish the validity of $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$ from the validity of Free Choice, which is much more intuitively compelling. In fact, our semantics can make this distinction. For Free Choice is not merely Strawson valid. It is also flat out truth preserving. So although both Free Choice and $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$ are both Strawson valid, the marginal appeal of Free Choice can still be explained.

Homogenous alternative semantics validates Free Choice, Disjunction Introduction, and Upwards Monotonicity. To escape Fact 1, this semantics gives up the transitivity of entailment. In particular, while $\Diamond\mathsf{A}$ implies $\Diamond(\mathsf{A} \vee \mathsf{B})$, and this last implies $\Diamond\mathsf{B}$, the premise $\Diamond\mathsf{A}$ does not by itself imply $\Diamond\mathsf{B}$.

Observation 5. $\Diamond\mathsf{A} \not\models \Diamond\mathsf{B}$

[13] See Asher and Bonevac 2005 for discussion.

Crucially, the first inference is only valid because we restrict attention to worlds where the conclusion $\diamondsuit(A \vee B)$ is defined. But this sentence appears nowhere in the argument from $\diamondsuit A$ to $\diamondsuit B$, and so this latter argument remains invalid.

We've now diagnosed how homogenous alternative semantics escapes the first incompatibility result from §1. Now let's turn to the other incompatibility results, involving Dual Prohibition. Here, the first point to notice is that for any proto-possibility operator $\Diamond$, $\diamondsuit$ satisfies Dual Prohibition.

Observation 6. $\neg\diamondsuit(A \vee B) \models \neg\diamondsuit A \wedge \neg\diamondsuit B$

When $\neg\diamondsuit(A \vee B)$ is true, $\Diamond!(A \vee B)$ is false. This implies that A and B are both impossible.

In §1, we saw that the joint validity of Free Choice and Dual Prohibition raises the danger of Explosion: that any two possibility claims are equivalent. In particular, given Transitivity and Contraposition, Free Choice and Dual Prohibition immediately imply Explosion. For by Free Choice, $\diamondsuit(A \vee B)$ implies $\diamondsuit B$; so by Contraposition $\neg\diamondsuit B$ implies $\neg\diamondsuit(A \vee B)$, which by Dual Prohibition implies $\neg\diamondsuit A$. The semantics above accepts each of the relevant instances of Contraposition.

Observation 7. $\neg\diamondsuit B \models \neg\diamondsuit(A \vee B)$.

Here again, the key is that the definedness of the conclusion requires A and B to be treated symmetrically.

To avoid Fact 2, homogenous alternative semantics gives up Transitivity. While $\neg\diamondsuit B$ implies $\neg\diamondsuit(A \vee B)$ and $\neg\diamondsuit(A \vee B)$ implies $\neg\diamondsuit A$, we do not have the first premise $\neg\diamondsuit B$ imply the last conclusion $\neg\diamondsuit A$. This is exactly parallel to our treatment of Fact 1 above. When we remove the intermediate step $\neg\diamondsuit(A \vee B)$, we are no longer entitled to restrict our attention to worlds where A and B are treated symmetrically.

Now let's turn to Fact 3. This showed that Free Choice and Dual Prohibition seem to require further revisions to the logic of disjunction, giving up either the Law of Excluded Middle or Constructive Dilemma. In particular, the concern was that without giving up one of these principles, we would be forced to accept the absurd 'Interconnectedness' principle IAT, that $(\diamondsuit A \wedge \diamondsuit B) \vee (\neg\diamondsuit A \wedge \neg\diamondsuit B)$ is a logical truth.

Our semantics offers the same solution to this problem as with Facts 1 and 2. First, LEM remains valid. In particular, we validate the potentially problematic instance involving Free Choice:

Observation 8. $\models \diamondsuit(A \vee B) \vee \neg\diamondsuit(A \vee B)$

Here, one thing to consider is that Strawson entailment considerably lowers the standards on being a logical truth. A logical truth doesn't need to always be defined; it simply needs to be true whenever defined. The instance above is only defined when A and B are homogenous, either both possible or both impossible. But when defined, the sentence is guaranteed to be true.[14]

Likewise, the semantics above validates Constructive Dilemma, including the instance used in the derivation of Fact 3. Here, the point is that applying Free Choice and Dual Prohibition to the two disjuncts above, we reach IAT.

[14]The last result is one place where our semantics differs from the treatment of homogeneity in Kriz 2015, who gives up the Law of Excluded Middle, accepting a generalization of a Strong Kleene logic rather than a supervaluationist one (see p. 47). Kriz 2015 motivates the denial of LEM on empirical grounds, suggesting that the following is not valid, since it is possible to deny:

Observation 9. $\Diamond(A \vee B) \vee \neg\Diamond(A \vee B) \models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$

Crucially, however, while the premise of the above is a logical truth, the conclusion is not. That is, we have another failure of Transitivity:

Observation 10. $\not\models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$

The IAT principle does not contain a disjunction under the scope of a possibility modal. So the IAT principle does not trigger a homogeneity effect. So in order for it to be a validity, it must be true at every world. But it fails at exactly the worlds where the LEM instance above is undefined: where A and B differ in modal status.

In this section, we have seen that homogenous alternative semantics provides an elegant treatment of the incompatibility results from §1. With each result, the semantics manages to validate all classical principles, with the exception of Transitivity. Most importantly, the semantics is able to deliver the joint validity of Free Choice and Dual Prohibition.

We've now developed a plausible strategy for validating Free Choice with homogeneity effects. In the second half of the paper, we will expand on this strategy by suggesting that homogeneity effects are a tool that a variety of theories of Free Choice can employ. In particular, we will show that in addition to alternative semantics, homogeneity effects can also be incorporated into dynamic semantics. In this case, we can let disjunction itself rather than possibility modals contribute the homogeneity effect, now requiring that either both or neither disjuncts are possible. This is not merely a proof of concept. It also has empirical advantages, concerning Free Choice effects where disjunction takes wide scope over possibility modals.

6 Homogenous dynamic semantics

In this section, we will explore the prospects for validating Free Choice and Dual Prohibition within dynamic semantics. In particular, we will look at one version of dynamic semantics: update semantics.[15] According to dynamic semantics, the meaning of a sentence is not its truth conditions. Rather, the meaning of a sentence is its ability to change the context in which it is said—its *context change potential*:

(i) Adam either read the books or he didn't read them.

(ii) Well, what if he read half of the books?

Interestingly, the analogous argument in our case is unsuccessful:

(iii) Either you can have soup or salad, or you can't.

(iv) ?Well, what if I can have soup but I can't have salad?

So perhaps the treatment in Kriz 2015 should be accepted for plurals, but not for Free Choice. On the other hand, nothing in the semantic clauses of Kriz 2015 prevents one from accepting Strawson validity, in which case that framework would also validate LEM.

[15]See Stalnaker 1973; Karttunen 1974; Heim 1982; Heim 1983; Veltman 1985; Groenendijk and Stokhof 1990; Groenendijk and Stokhof 1991; and many others.

> You know the meaning of a sentence if you know the change it brings about in the information state of anyone who accepts the news conveyed by it.[16]

To give an update semantics, we need two things: a definition of information states (or contexts), and an interpretation function which assigns a context change potential to each sentence in our language. Veltman 1996 models an information state as a set of possible worlds. Then an interpretation function assigns every sentence a context change potential—a function from sets of worlds to sets of worlds. We will also be making use of definedness conditions on updating, so that our context change potentials can be partial.

Definition 12. A possible world w assigns every atomic sentence α a truth value. W is the set of all possible worlds. An information state s is a set of possible worlds. A context change potential is a partial function from one information state s to a new one. An interpretation function $[\cdot]$ assigns every sentence a context change potential. $s[\mathsf{A}]$ is the result of inputting s into $[\mathsf{A}]$.

Once we have a representation of information states, we can then recursively define our interpretation function, $[\cdot]$. Here, we will assume that atomic sentences simply narrow down an information state to the worlds where they are true.

Definition 13. $s[\mathsf{p}] = \{w \in s \mid w(\mathsf{p}) = 1\}$

Next, we can hold fixed the usual dynamic semantics for negation. On this proposal, updating an information state with $\neg\mathsf{A}$ returns exactly the worlds that would not survive updating with A. As in §2 however, we will be making use of undefinedness in what follows. To get the right results, we will model undefinedness as in Heim 1992, in terms of the partiality of updating. In particular, we will say that an update with $\neg\mathsf{A}$ is only defined when an update with A is defined.

Definition 14. $s[\neg\mathsf{A}]$ is defined only if $s[\mathsf{A}]$ is defined.
If defined, $s[\neg\mathsf{A}] = s - s[\mathsf{A}]$

Atomic sentences and their negations are in the business of narrowing down the worlds in an information state. By contrast, modal sentences don't give us new information about what world we are in. Instead, they are tests, exploring properties of the current state. $\Diamond\mathsf{A}$ explores whether s can be consistently updated with A. If so, the initial state is unchanged by updating. Otherwise, the absurd state $\emptyset$ results. Again, however, we will be allowing some updates in this system to be undefined. To get the right predictions about Free Choice, we will assume that updating with $\Diamond\mathsf{A}$ is undefined when updating with A is undefined.

Definition 15. $s[\Diamond\mathsf{A}]$ is defined only if $s[\mathsf{A}]$ is defined.
If defined, $s[\Diamond\mathsf{A}] = \begin{cases} s & \text{if } s[\mathsf{A}] \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$

Before turning to disjunction, we also need a definition of entailment. Say that a state supports a sentence A just in case updating the state with A has no effect. Then we can

[16] See Veltman 1996, p. 221.

introduce a dynamic version of Strawson entailment, which says that an argument is valid just in case any state where the premises are supported and the conclusion is defined is a state which supports the conclusion.[17]

Definition 16.

1. s supports A ($s \models \mathsf{A}$) iff $s[\mathsf{A}] = s$.
2. Some premises $\mathsf{A}_1, \ldots \mathsf{A}_n$ entail C just in case $s \models \mathsf{C}$ whenever:
   (a) $s \models \mathsf{A}_1; \ldots; s \models \mathsf{A}_n$.
   (b) $s[\mathsf{C}]$ is defined.

With our assumptions in place, we can turn to Free Choice. To validate this inference, we can use one simple idea: disjunctions require each disjunct to be possible.[18] That is, we need a semantics on which the following inference is valid:

MODAL DISJUNCTION $\mathsf{A} \vee \mathsf{B} \models \Diamond\mathsf{A}$ and $\mathsf{A} \vee \mathsf{B} \models \Diamond\mathsf{B}$

At first glance, Modal Disjunction looks like a bold thesis.[19] But on further reflection, we can see that there is an intimate connection between Modal Disjunction and the validity of Free Choice. To see why, note that the dynamic semantics for possibility modals above validates the T axiom, which says that anything true is possible:

T $\mathsf{A} \models \Diamond\mathsf{A}$

Given the duality of *might* and *must*, the T axiom is equivalent to the requirement that *must* is strong, so that $\Box\mathsf{A}$ implies A.[20]

Modal Disjunction then follows from Free Choice, holding fixed the T axiom and the transitivity of entailment. After all, the T axiom implies that $\mathsf{A} \vee \mathsf{B} \models \Diamond(\mathsf{A} \vee \mathsf{B})$. By Free Choice, this immediately implies $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$. (As usual, homogenous alternative semantics avoids this result by giving up the Transitivity. By contrast, an ordinary alternative semantics for Free Choice avoids this argument by rejecting T, so that $\mathsf{A} \vee \mathsf{B}$ does not imply $\Diamond(\mathsf{A} \vee \mathsf{B})$.)

We just saw that Free Choice implies Modal Disjunction, given a few assumptions. Similarly, given a few more assumptions Modal Disjunction implies Free Choice. For suppose we accept the 4 axiom, so that anything possibly possible is itself possible:

4 $\Diamond\Diamond\mathsf{A} \models \Diamond\mathsf{A}$

[17] See Veltman 1996 and van Benthem 1996 139-41 for discussion of how to define entailment in dynamic semantics. Our results below could also be established using a Strawsonian version of update-to-test entailment.

[18] See Zimmermann 2000 and Geurts 2005 for other implementations of this idea.

[19] For an overview of Modal Disjunction, see Cariani 2017.

[20] For discussion of whether this principle is valid for epistemic modals, see among others Karttunen 1972; Kratzer 1991; von Fintel and Gillies 2010; and Lassiter 2016.

If we combine the 4 axiom with the Upwards Monotonicity of $\Diamond$, Modal Disjunction implies Free Choice.[21] By Modal Disjunction, $\mathsf{A} \vee \mathsf{B}$ implies $\Diamond\mathsf{B}$. So by Upwards Monotonicity, $\Diamond(\mathsf{A} \vee \mathsf{B})$ implies $\Diamond\Diamond\mathsf{B}$, which by the 4 axiom implies $\Diamond\mathsf{B}$. So, summing up, if our possibility operators satisfy the T and 4 axioms, then Free Choice and Modal Disjunction are equivalent.

Fact 4. Suppose Transitivity, Upwards Monotonicity, T and 4. Then Free Choice is valid iff Modal Disjunction is valid.

Our task now is to provide a dynamic semantics for disjunction on which Modal Disjunction and Free Choice are valid. Here is where we can again appeal to homogeneity. We can say that the disjunction $\mathsf{A} \vee \mathsf{B}$ is defined only when A and B share the same modal status. Given our dynamic framework, this means that either the information state can be consistently updated with each of A and B, or it can be consistently updated with neither of them. Finally, when defined, $[\mathsf{A} \vee \mathsf{B}]$ then narrows down the state to worlds where one of A or B is true.

Definition 17. $s[\mathsf{A} \vee \mathsf{B}]$ is defined only if either:

1. $s \models \Diamond\mathsf{A}$ and $s \models \Diamond\mathsf{B}$, or
2. $s \models \neg\Diamond\mathsf{A}$ and $s \models \neg\Diamond\mathsf{A}$

If defined, $s[\mathsf{A} \vee \mathsf{B}] = s[\mathsf{A}] \cup s[\mathsf{B}]$.

This theory of disjunction incorporates homogeneity effects into the normal dynamic theory of disjunction, according to which $\mathsf{A} \vee \mathsf{B}$ unions together the result of updating with each of A and B. When A and B are non-modal, this results in classical truth conditions for disjunction, so that updating an information state with $\mathsf{A} \vee \mathsf{B}$ narrows down the state to the worlds where one of A and B are true. Now, however, we've enriched this idea with a homogeneity effect, so that $\mathsf{A} \vee \mathsf{B}$ is only defined at information states that contain both A worlds and B worlds, or contain neither A nor B worlds. With all of this in place, we can now turn to the puzzles from §1.

7 Results

Free Choice is valid in homogenous dynamic semantics.

Observation 11. $\Diamond(\mathsf{A} \vee \mathsf{B}) \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$

Whenever s supports $\Diamond(\mathsf{A} \vee \mathsf{B})$, $s[\mathsf{A} \vee \mathsf{B}]$ is defined. This in turn requires that either s supports both $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$, or neither. But since s supports $\Diamond(\mathsf{A} \vee \mathsf{B})$, we also know that $s[\mathsf{A}] \cup s[\mathsf{B}]$ is non-empty. So s supports both $\Diamond\mathsf{A}$ and $\Diamond\mathsf{B}$.

Since homogenous dynamic semantics validates Free Choice, it must also respond to Fact 1, concerning the classical properties of disjunction. As in homogenous alternative semantics, Disjunction Introduction is valid, although for a different reason.

[21] See Zimmermann 2000 for a similar observation.

Observation 12. $\mathsf{A} \models \mathsf{A} \vee \mathsf{B}$

Here, the proof relies on Strawson validity, which allows us to restrict our attention to information states in which $\mathsf{A} \vee \mathsf{B}$ is defined. When any such state supports A, it also supports $\mathsf{A} \vee \mathsf{B}$, since we can ignore the possibility that *s* contains no Bworlds.

This system also shares another property with homogenous alternative semantics: Upwards Monotonicity is valid. Of particular interest for Fact 1: this theory validates the instance of Upwards Monotonicity that uses Disjunction Introduction.

Observation 13. $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$

As in homogenous alternative semantics, the definedness of the conclusion requires that either both or neither of A and B are possible. Combined with the support of the premise $\Diamond\mathsf{A}$, this means that $\Diamond\mathsf{B}$ is supported, and so $\Diamond(\mathsf{A} \vee \mathsf{B})$ is supported.

Again, as in homogenous alternative semantics this theory also allows us to distinguish the validity of $\Diamond\mathsf{A} \models \Diamond(\mathsf{A} \vee \mathsf{B})$ from the validity of Free Choice, which is more compelling. Free Choice is not merely Strawson valid; it is also flat out support preserving.

Homogenous dynamic semantics validates Free Choice, Disjunction Introduction, and Upwards Monotonicity. To escape Fact 1, this semantics also gives up the transitivity of entailment. While $\Diamond\mathsf{A}$ implies $\Diamond(\mathsf{A} \vee \mathsf{B})$, and this last implies $\Diamond\mathsf{B}$, the premise $\Diamond\mathsf{A}$ does not by itself imply $\Diamond\mathsf{B}$.

Observation 14. $\Diamond\mathsf{A} \not\models \Diamond\mathsf{B}$

Again, the first inference is only valid because we restrict attention to information states where the conclusion $\Diamond(\mathsf{A} \vee \mathsf{B})$ is defined. But this sentence appears nowhere in the argument from $\Diamond\mathsf{A}$ to $\Diamond\mathsf{B}$, and so this latter argument remains invalid.

Homogenous dynamic semantics has so far had the same results as homogenous alternative semantics. The two frameworks differ, however, in the features of bare disjunctions. Homogenous dynamic semantics validates not only Free Choice but also Modal Disjunction:

Observation 15. $\mathsf{A} \vee \mathsf{B} \models \Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$.

We saw with Fact 4 that Modal Disjunction is equivalent to Free Choice given Upwards Monotonicity, the T axiom, the 4 axiom, and Transitivity. All of the above hold in this system except Transitivity. But while Transitivity fails in general, the particular instances relevant to Fact 4 do not fail.[22]

We've now diagnosed how homogenous dynamic semantics escapes the first incompatibility result from §1. Now let's turn to the other incompatibility results, involving Dual Prohibition. First, one of the signature properties of this system is that Dual Prohibition is valid:

Observation 16. $\neg\Diamond(\mathsf{A} \vee \mathsf{B}) \models \neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B}$

[22] We suspect that the basic ideas in homogenous dynamic semantics could also be extended to other modal theories of disjunction. For example, in Zimmermann 2000, $\mathsf{A} \vee \mathsf{B}$ is interpreted directly as a list of possibility claims. This could be enriched with a homogeneity effect in a similar way as in our own theory.

Whenever $\neg\Diamond(A \vee B)$ is supported, it is also defined. When it is defined, either A and B are both possible or both impossible. So if $\neg\Diamond(A \vee B)$ is supported, both A and B are impossible.

In §1, we saw that the joint validity of Free Choice and Dual Prohibition increases the danger of Explosion, where any two possibility claims are equivalent. In particular, given Transitivity and Contraposition, Free Choice and Dual Prohibition immediately imply Explosion. For by Free Choice, $\Diamond(A \vee B)$ implies $\Diamond B$; so by Contraposition $\neg\Diamond B$ implies $\neg\Diamond(A \vee B)$, which by Dual Prohibition implies $\neg\Diamond A$.

A first natural reaction to this result is to note that homogenous dynamic semantics gives up Contraposition in general. As in ordinary versions of update semantics, A implies $\Box A$ (letting $\Box$ be the dual of $\Diamond$), but $\Diamond A$ does not imply A. Nonetheless, this failure of Contraposition is not relevant to our second impossibility result. The semantics above accepts each of the relevant instances of Contraposition in that result.

Observation 17. $\neg\Diamond B \models \neg\Diamond(A \vee B)$

Here again, the key is that the definedness of the conclusion requires A and B to be treated symmetrically.

As in homogenous alternative semantics, this theory avoids Fact 2 by giving up Transitivity. While $\neg\Diamond B$ implies $\neg\Diamond(A \vee B)$ and $\neg\Diamond(A \vee B)$ implies $\neg\Diamond A$, we do not have the first premise $\neg\Diamond B$ imply the last conclusion $\neg\Diamond A$. This is exactly parallel to our treatment of Fact 1 above. When we remove the intermediate step $\neg\Diamond(A \vee B)$, we are no longer entitled to restrict our attention to states where A and B are treated symmetrically.

Now let's turn to Fact 3, which placed Free Choice and Dual Prohibition in tension with the Law of Excluded Middle and Constructive Dilemma. Again, the problem was that all together these principles implied IAT, that $(\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$ is a logical truth.

Here, our semantics uses the same solution strategy as with Facts 1 and 2. First, LEM remains valid, including the following instance:

Observation 18. $\models \Diamond(A \vee B) \vee \neg\Diamond(A \vee B)$

As in homogenous alternative semantics, the key observation is that Strawson entailment requires much less of logical truths. Rather than support at all states, we only require the sentence to be supported at the states in which it is defined. The above state is only defined at states which either contain both A worlds and B worlds, or which contain neither kind of worlds. At any such state, the test imposed by one of the disjuncts is passed. Likewise, the semantics above validates Constructive Dilemma, including the instance used in the derivation of Fact 3. Here, the point is that applying Free Choice and Dual Prohibition to the two disjuncts above, we reach IAT.

Observation 19. $\Diamond(A \vee B) \vee \neg\Diamond(A \vee B) \models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$

Crucially, while the premise of the above is a logical truth, the conclusion is not. That is, we again have another failure of Transitivity:

Observation 20. $\not\models (\Diamond A \wedge \Diamond B) \vee (\neg\Diamond A \wedge \neg\Diamond B)$

IAT fails spectacularly in this system: it is not only invalid, but inconsistent. That is: whenever it is defined, its negation is supported.[23] For the same reason, Observation 19 only holds vacuously, because there is no state where the conclusion is defined and the premise is true.

One might worry that homogenous dynamic semantics rejects IAT in too strong a fashion. Although IAT should not be a validity, it should be consistent. In the next section, we will generalize the system so far to account for other flavors of modals, like deontic modals. This generalization will predict that IAT is consistent.

In this section, we have seen that homogenous dynamic semantics provides an elegant treatment of the logical incompatibility results from §1. With each result, the semantics manages to validate all classical principles, with the exception of Transitivity. Most importantly, the semantics is able to deliver the joint validity of Free Choice and Dual Prohibition. So far, however, the reader may have a concern. Homogenous dynamic semantics seems tailor-made for epistemic versions of Free Choice. But how can it extend to Free Choice for other flavors of modality? In the next section we will see how to achieve this result.

8 Extension to other modals

So far, we've focused our discussion on epistemic modals. Here, we'll achieve similar results for other flavors of modals. The upshot will be that we can treat modals in a truth conditional way and still reap the benefits of the above, provided that disjunctions themselves use a dynamic kind of possibility operator.

Above, we appealed to possibility operators in two separate ways. First, Free Choice involved embedding disjunction under some sort of possibility modal. Second, our semantics had disjunctions themselves require that each disjunct be possible. We'll now see that these two appeals to possibility modals can be treated separately. In particular, we will see that we can integrate a static semantics for deontic modals with our dynamic semantics for disjunction in a way that will validate Free Choice for deontic modals.

In particular, we can incorporate a possible worlds semantics for modals into our dynamic framework in a straightforward way. We can introduce a possible worlds modal $\diamondsuit$ into this system that is parameterized to an underlying accessibility relation R, so that $\diamondsuit \mathsf{A}$ narrows down an information state to the worlds whose R-accessible possibilities can be consistently updated with A:

Definition 18. $s[\diamondsuit \mathsf{A}] = \big\{ w \in s \mid \{v \mid wRv\}[\mathsf{A}] \neq \emptyset \big\}$

This semantics for modals allows us to model arbitrary flavors of modality. For example, we can model Free Choice for deontic modals by appeal to a deontic accessibility relation

[23] To see why, note first that IAT is only defined in states in which one of A and B is possible, and the other is not. For IAT is defined in s only if either (i) s supports both $\Diamond(\Diamond \mathsf{A} \wedge \Diamond \mathsf{B})$ and $\Diamond(\neg\Diamond \mathsf{A} \wedge \neg\Diamond \mathsf{B})$, or (ii) s supports $\neg\Diamond(\Diamond \mathsf{A} \wedge \Diamond \mathsf{B})$ and $\neg\Diamond(\neg\Diamond \mathsf{A} \wedge \neg\Diamond \mathsf{B})$. But the former condition is unsatisfiable, because in this system iterated modals collapse, and so condition (i) is equivalent to the absurd requirement that both $\Diamond \mathsf{A} \wedge \Diamond \mathsf{B}$ and $\neg\Diamond \mathsf{A} \wedge \neg\Diamond \mathsf{B}$ be supported. So IAT is defined only if condition (ii) holds, so that: $\neg\Diamond(\Diamond \mathsf{A} \wedge \Diamond \mathsf{B})$ and $\neg\Diamond(\neg\Diamond \mathsf{A} \wedge \neg\Diamond \mathsf{B})$. This in turn can hold only if one of *na* and B is possible, and another is not.

But now suppose that s makes one of A and B possible, and the other impossible. Then the ordinary update of IAT is guaranteed to fail. For updating s with either disjunct of IAT produces the absurd state; and so the union of these updates is absurd.

R_d, which relates any world w to the worlds that are consistent with the normative rules at w.

Now we can retain our earlier semantics for disjunction, defined in terms of the test operator $\Diamond$. But we can allow this dynamic disjunction operator to embed under $\Diamonddot$. The resulting theory validates Free Choice:

Observation 21. $\Diamonddot(\mathsf{A} \vee \mathsf{B}) \models \Diamonddot\mathsf{A} \wedge \Diamonddot\mathsf{B}$

For s supports $\Diamonddot(\mathsf{A} \vee \mathsf{B})$ only if for every world w in s, $\{v \mid wRv\}[\mathsf{A} \vee \mathsf{B}] \neq \emptyset$. But given the semantics for $\vee$, this in turn requires that $\{v \mid wRv\} \models \Diamond\mathsf{A}$ and $\{v \mid wRv\} \models \Diamond\mathsf{B}$, so that $\{v \mid wRv\}[\mathsf{A}] \neq \emptyset$ and $\{v \mid wRv\}[\mathsf{B}] \neq \emptyset$. Since w is arbitrary in s, this implies that $s \models \Diamonddot\mathsf{A}$ and $s \models \Diamonddot\mathsf{B}$.

To summarize, the crucial property needed here is that the following principle is valid:

OUTER POSITIVE INTROSPECTION $\Diamonddot\Diamond\mathsf{A} \models \Diamonddot\mathsf{A}$.

The reason that we are able to validate this principle regardless of the structure of R is that $\Diamond$ is a dynamic operator. $\Diamond\mathsf{A}$ predicates possibility of a body of information. But because $\Diamond\mathsf{A}$ is dynamic, it can in principle predicate possibility of different bodies of information, depending on the environment in which it occurs. So if $\Diamond\mathsf{A}$ applies to the whole information state s, $\Diamond\mathsf{A}$ will predicate epistemic possibility of A—consistency with what is common knowledge, say. By contrast, when $\Diamond\mathsf{A}$ is embedded under an information shifting operator, like $\Diamonddot$, it has a different effect. Under $\Diamonddot$, $\Diamond\mathsf{A}$ says that A is consistent with $\{v \mid wRv\}$. That is, $\Diamond$ says the same thing under the scope of $\Diamonddot$ as $\Diamonddot$ says when unembedded. This sensitivity to local context is a hallmark of dynamic semantics.

We've now seen that a truth conditional semantics for possibility modals can be integrated into our dynamic semantics for disjunction in a way that also validates Free Choice. Similarly, this revised semantics validates Dual Prohibition. More generally, this semantics has the same results as that in the previous section, with one exception. In the previous section, we predicted that IAT ($[\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}] \vee [\neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B}]$) was not only invalid, but inconsistent. This seemed too strong, since for example (16) seems like it could be true:

(16) Mary may have soup and Mary may have salad; or Mary may not have soup and Mary may not have salad.

Here, our new semantics makes just the right prediction. IAT is still invalid, but is also consistent:

Observation 22. $(\Diamonddot\mathsf{A} \wedge \Diamonddot\mathsf{B}) \vee (\neg\Diamonddot\mathsf{A} \wedge \neg\Diamonddot\mathsf{B}) \not\models \bot$

For example, consider a state made up of two worlds. At the first world, A and B are both deontically permissible. At the second world, neither A nor B are deontically permissible. This state supports $(\Diamonddot\mathsf{A} \wedge \Diamonddot\mathsf{B}) \vee (\neg\Diamonddot\mathsf{A} \wedge \neg\Diamonddot\mathsf{B})$, because each disjunct holds at one of the two worlds.

9 Comparisons

We've now developed two different semantics that use homogeneity to validate Free Choice and Dual Prohibition. In this section, we'll explore two factors that might decide between these two systems.

9.1 Wide free choice

Free Choice occurs not only when possibility modals scope over disjunction, but also when disjunction at least appears to take wide scope to possibility modals.[24] For example, (17) seems to imply (18):

(17) Mary might be in New York or she might be in Los Angeles.

(18) Mary might be in New York and she might be in Los Angeles.

WIDE FREE CHOICE $\Diamond \mathsf{A} \vee \Diamond \mathsf{B} \models \Diamond \mathsf{A} \wedge \Diamond \mathsf{B}$

Homogenous alternative semantics and homogenous dynamic semantics differ with respect to this inference.

First, this inference is invalid in homogenous alternative semantics. Here, the key is that Wide Free Choice never allows an alternative-denoting sentence to scope below a possibility modal. For this reason, no homogeneity effects are triggered by Wide Free Choice in this system. So $\Diamond \mathsf{A} \vee \Diamond \mathsf{B}$ is always defined, and true at a world just in case there is some accessible A world or some accessible B world.

By contrast, Wide Free Choice is valid in homogenous dynamic semantics. In homogenous dynamic semantics, homogeneity effects are contributed by disjunction itself. So $\Diamond \mathsf{A} \vee \Diamond \mathsf{B}$ is itself defined in s only if s supports $\Diamond\Diamond \mathsf{A}$ and s supports $\Diamond\Diamond \mathsf{B}$. Since $\Diamond$ satisfies the 4 axiom, Wide Free Choice is valid for the epistemic test operator $\Diamond$.

Things are a bit more complicated for versions of Wide Free Choice with other modals. In homogenous dynamic semantics, this involves the inference from $\dot{\Diamond} \mathsf{A} \vee \dot{\Diamond} \mathsf{B}$ to $\dot{\Diamond} \mathsf{A} \wedge \dot{\Diamond} \mathsf{B}$. Unlike the narrow scope version, this inference's validity turns on properties of the accessibility relation associated with $\dot{\Diamond}$. In particular, the inference is valid only if R is universal, so that whenever v is accessible from any w, v is accessible from every w. For suppose v is accessible from w but not u. Now let $s = \{w, u\}$, and let A be a claim true at v uniquely. s supports $\Diamond\dot{\Diamond} \mathsf{A}$, since $s[\dot{\Diamond} \mathsf{A}] = \{w\} \neq \emptyset$. But s does not support $\dot{\Diamond} \mathsf{A}$, since s contains u, which cannot see v.

For this reason, homogenous dynamic semantics allows that Wide Free Choice is cancellable for deontic, but not epistemic modals.[25]

(19) You may take an apple or you may take a pear, but I don't know which.

(20) You make take an apple or you may take a pear, but I won't tell you which.[26]

In homogenous dynamic semantics, any case with this form involves a modal being interpreted relative to a non-universal accessibility relation. Finally, Simons 2005 suggests there may be a few cases in which Free Choice can be cancelled even for epistemic modals. We can explain these cases by proposing that sometimes the relevant epistemic modal is not the test operator $\Diamond$, but rather $\dot{\Diamond}$ under some kind of epistemic accessibility relation.

Finally, in homogenous dynamic semantics there is no analogous way to cancel narrow scope Free Choice inferences. These are valid regardless of the choice of accessibility

[24] See Zimmermann 2000 for a semantic account of Wide Free Choice, Simons 2005 for an argument that apparent instances of Wide Free Choice actually involve narrow scope disjunction, and Willer 2017b for a pragmatic account of Wide Free Choice.

[25] A similar prediction is achieved in Zimmermann 2000.

[26] See Kamp 1978 and Willer 2017b for discussion.

relation. But the narrow scope analogues of (19) and (20) (the sentences (8) and (9) from §2) are equally compelling. We can then explain these cases by suggesting that at the level of logical form they have disjunction taking wide scope to $\Diamond$, with a non-universal accessibility relation.

It is no coincidence that homogenous dynamic semantics validates Wide Free Choice. We know that homogenous dynamic semantics is a modal theory of disjunction. But it turns out that Wide Free Choice is intimately connected to Modal Disjunction. Given a variety of principles we have already explored, these two principles are equivalent.

Fact 5. Suppose Transitivity, T, the 4 axiom and Constructive Dilemma. Then Wide Free Choice is valid iff Modal Disjunction is valid.

Proof. For the left to right direction: by T, A implies $\Diamond\mathsf{A}$ and B implies $\Diamond\mathsf{B}$. So by Constructive Dilemma $\mathsf{A} \vee \mathsf{B}$ implies $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$. So by Transitivity and Wide Free Choice, $\mathsf{A} \vee \mathsf{B}$ implies $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$. For the right to left direction: by Modal Disjunction, $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$ implies $\Diamond\Diamond\mathsf{A} \wedge \Diamond\Diamond\mathsf{B}$. By the 4 axiom, $\Diamond\Diamond\mathsf{A} \wedge \Diamond\Diamond\mathsf{B}$ implies $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$. So by Transitivity, $\Diamond\mathsf{A} \vee \Diamond\mathsf{B}$ implies $\Diamond\mathsf{A} \wedge \Diamond\mathsf{B}$. □

Together, Facts 4 and 5 also create a bridge from the validity of Free Choice to the validity of Wide Free Choice. Given the T axiom and Transitivity, we can infer Modal Disjunction from Free Choice. Given the 4 axiom and Transitivity, we can infer Wide Free Choice from Modal Disjunction, and hence from Free Choice. More generally, if we hold fixed all of the assumptions from Facts 4 and 5, Free Choice and Wide Free Choice are equivalent.

Just like Free Choice, Wide Free Choice also disappears under negation. To test for this effect, we can embed instances of Wide Free Choice under negative attitude verbs like *doubts*, which are intuitively equivalent to *believes not*. For example, (21) appears to entail (22):

(21) Alex doubts that Billie might be in New York or Billie might be in Los Angeles.

(22) Alex believes that Billie can't be in New York and Billie can't be in Los Angeles.

WIDE DUAL PROHIBITION $\neg(\Diamond\mathsf{A} \vee \Diamond\mathsf{B}) \models \neg\Diamond\mathsf{A} \wedge \neg\Diamond\mathsf{B}$

Homogenous dynamic semantics handles this data with ease. A state supports $\neg(\Diamond\mathsf{A} \vee \Diamond\mathsf{B})$ only if either both A and B are possible, or both are impossible. The ordinary update of $\neg(\Diamond\mathsf{A} \vee \Diamond\mathsf{B})$ rules out the former condition; so both must be impossible.

So far, we've considered one advantage of homogenous dynamic semantics over homogenous alternative semantics.[27] Now we will turn to one potential disadvantage.

[27] Another potential advantage of homogenous dynamic semantics over homogenous alternative semantics concerns mixed modal effects. For example, Simons 2005 observes that (i) appears equivalent to (ii):

(i) Mary may sing, or Harriet must dance.

(ii) Mary may sing and if she doesn't, then Harriet must dance.

This can be explained in homogenous dynamic semantics, provided that we allow disjunction to introduce a more sophisticated kind of update:

Definition 19. $s[\mathsf{A} \vee \mathsf{B}]$ is defined only if either (i) $s[\mathsf{A}] \neq \emptyset$ and $[\neg\mathsf{A}][\mathsf{B}] \neq \emptyset$; or (i) $s[\mathsf{A}] = \emptyset$ and

9.2 Quantifiers

Homogenous dynamic semantics, and modal theories of disjunction in general, face a *prima facie* difficulty when disjunction is embedded under quantifiers. For example, consider (23):

(23) Every philosopher or linguist went to the party.

One might worry that homogenous dynamic semantics makes a bad prediction about (23): that it only quantifies over people who might be a philosopher and also might be a linguist. After all, on the account above the open formula *x is a philosopher or x is a linguist* should be defined only if either both claims are possible, or both are impossible. Intuitively, however, (23) is false if someone who must be a linguist and not a philosopher did not go to the party.

Of course, to understand this prediction we have to integrate the dynamic semantics above with a semantics for quantification. I'll now argue that when we do so, the problem above disappears. If we extend our semantics in the right way, we can predict that the disjunctive restrictor in (23) quantifies over all philosophers and all linguists.[28]

Here we'll consider a semantics for quantifiers that is a simplification of the dynamic semantics in Groenendijk et al. 1996. For brevity, we'll omit many details of the semantics; but a full implementation can be found in Carter and Goldstein 2017. One of the goals of this semantics is to make the right predictions about donkey anaphora, so that (24) and (25) are equivalent:

(24) If a donkey is beaten, it is sad.

(25) Every donkey that is beaten is sad.

We'll model universal quantification using a generalized quantifier $\forall\mathsf{x}$ that takes two sentences as input, and we'll model indefinites like *a donkey* with a special operator $\exists\mathsf{x}$, which occurs within the antecedent of the conditional in (24). This gives us:

REDUCTION $\exists\mathsf{x}\mathsf{F}\mathsf{x} \to \mathsf{G}\mathsf{x} =\!\mid\models \forall\mathsf{x}(\mathsf{F}\mathsf{x})(\mathsf{G}\mathsf{x})$

Our semantics will enrich contexts to be sets of assignment-world pairs rather than just sets of worlds. Then $[\mathsf{F}\mathsf{x}]$ narrows down the context to the assignment-world pairs where the referent of x satisfies F. Crucially, $[\exists\mathsf{x}]$ performs a non-eliminative update, expanding the context so that any $\langle g, w\rangle$ pair in the current context is joined by any pair $\langle g', w\rangle$ where g and g' differ only in the value of x. The operators $\to$ and $\Diamond$ retain their usual test semantics, so that $\mathsf{A} \to \mathsf{C}$ tests that the context is unchanged by Cafter being updated with A. Finally, we can simply define the sentence $\forall\mathsf{x}(\mathsf{A})(\mathsf{B})$ directly in terms of $\exists\mathsf{x}$ and $\to$, to achieve Reduction by brute force.

Definition 20.

1. $c[\mathsf{F}\mathsf{x}] = \{\langle g, w\rangle \in c \mid g(\mathsf{x}) \in V(\mathsf{F})(w)\}$

$[\neg\mathsf{A}][\mathsf{B}] = \emptyset$.
If defined, $s[\mathsf{A} \vee \mathsf{B}] = s[\mathsf{A}] \cup s[\neg\mathsf{A}][\mathsf{B}]$

[28]This is related to some observations in Büring 1998.

2. $c[\exists \mathsf{x}] = \{\langle g', w\rangle \mid g[x]g' \;\&\; \langle g, w\rangle \in c\}$
3. $c[\exists \mathsf{x}\mathsf{A}] = c[\exists \mathsf{x}][\mathsf{A}]$
4. $c[\mathsf{A} \vee \mathsf{B}]$ is defined only if either (i) $c[\mathsf{A}] \neq \emptyset$ and $c[\mathsf{B}] \neq \emptyset$ or (ii) $c[\mathsf{A}] = \emptyset$ and $c[\mathsf{B}] = \emptyset$.

   If defined, $c[\mathsf{A} \vee \mathsf{B}] = c[\mathsf{A}] \cup c[\mathsf{B}]$
5. $c[\mathsf{A} \rightarrow \mathsf{C}] = \begin{cases} c & \text{if } c[\mathsf{A}] \models \mathsf{C} \\ \emptyset & \text{otherwise} \end{cases}$
6. $c[\Diamond \mathsf{A}] = \begin{cases} c & \text{if } c[\mathsf{A}] \neq \emptyset \\ \emptyset & \text{otherwise} \end{cases}$
7. $c[\forall \mathsf{x}(\mathsf{A})(\mathsf{C})] = c[\exists \mathsf{x}\mathsf{A} \rightarrow \mathsf{C}]$

With a simple dynamic theory of quantifiers and modals in place, let's return to the meaning of disjunctive quantified claims like $\forall \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx})(\mathsf{Hx})$. This sentence has the same meaning as $\exists \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx}) \rightarrow \mathsf{Hx}$. The key property here is that $c[\exists \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx}) \rightarrow \mathsf{Hx}]$ explores whether $c[\exists \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx})]$ is unchanged by $[\mathsf{Hx}]$. Now $c[\exists \mathsf{x}(\mathsf{Fx} \vee \mathsf{Gx})]$ is in turn decomposed into $c[\exists \mathsf{x}][(\mathsf{Fx} \vee \mathsf{Gx})]$. $c[\exists x]$ expands c to allow x to refer to any individual. $c[\exists \mathsf{x}][(\mathsf{Fx} \vee \mathsf{Gx})]$ is then defined only if $c[\exists \mathsf{x}][\mathsf{Fx}]$ and $c[\exists \mathsf{x}][\mathsf{Gx})]$ are each non-empty. This requires that $c[\exists \mathsf{x}]$ include some $\langle g, w\rangle$ where $g(\mathsf{x})$ is F at w, and include some $\langle g', w'\rangle$ where $g'(\mathsf{x})$ is G at w'. Since $c[\exists \mathsf{x}]$ includes any possible value of x, this reduces to the requirement that c include some world where something is *F*, and some world where something is *G*. In other words, the disjunctive restrictor only applies a possibility test to the global context; it does not place this requirement on each individual one at a time. Finally, $c[\exists \mathsf{x}][(\mathsf{Fx} \vee \mathsf{Gx})]$ narrows down $c[\exists \mathsf{x}]$ to the assignment-world pairs where x is either F or G. Summarizing, the whole sentence is then supported by a context just in case (i) the context contains a world where something is F, and a world where something is G; and (ii) every individual who is F or G at any world in the context is also H. This is a perfectly reasonable prediction, and avoids the concerns above. When someone who must be a linguist and not a philosopher fails to go to the party, condition (ii) fails and (23) is false.

Summarizing, then, we've seen that if we have a sophisticated treatment of quantification in dynamic semantics, we can allow disjunction to be modal without making bad predictions about the way in which disjunction embeds under quantifiers.

10 Conclusion

In this paper, I've shown that there is a natural way to enrich either alternative semantics or dynamic semantics with homogeneity effects in order to validate both Free Choice and Dual Prohibition simultaneously. This provides significant support for a semantic account of Free Choice, since it undercuts one of the main objections to such a strategy.

It turns out that there are other reasons to want this kind of theory. Building on Santorio 2017, Cariani and Goldstein 2017 develop a version of homogenous alternative semantics for conditionals in order to validate another combination of principles that appear jointly inconsistent. They develop a battery of incompatibility results, showing that Simplification of Disjunctive Antecedents is in *prima facie* conflict with Conditional Excluded Middle. Then using homogenous alternative semantics, they validate both principles, by giving up the transitivity of entailment. Together with the present paper, homogenous alternative semantics thus satisfies Willer 2017a's goal of providing a unified solution to Free Choice and Simplification of Disjunctive Antecedents.

Putting these papers together, it looks like the synthesis of homogeneity and alternatives (or dynamics) provides a powerful tool with which to combat various results showing that within classical logic various constellations of plausible principles are jointly incompatible.

Of course, all of this came with one major cost: rejecting the transitivity of entailment. If entailment is understood as necessary truth-preservation (which of course is not how we characterized it), Transitivity should be a basic property. How much do we give up by moving to an intransitive notion of entailment? And is it worth it?

One caveat for the defender of Transitivity is that within our framework, violations of Transitivity are highly localized. Recall our formulation of Transitivity:

TRANSITIVITY If $\mathsf{A} \models \mathsf{B}$ and $\mathsf{B} \models \mathsf{C}$, then $\mathsf{A} \models \mathsf{C}$

Generally speaking: Transitivity can only fail, in the context of Strawson entailment, if B plays an essential role in satisfying the definedness conditions on C. As a consequence, various restricted forms of Transitivity are unaffected. Specifically, there are no violations of Transitivity when C lacks definedness conditions.

Depending on what we think motivates Transitivity, these restricted forms might be all we need.[29] If we thought that these generalizations are supported by the intuitive plausibility of their instances, we should not be troubled by the retreat to the restricted forms of the rules. The cases in which Transitivity fails are not only relatively localized, but they are also not obviously cases in which Transitivity is supported.

[29]There is much prior work on restrictions of Transitivity, although generally not in the context of Strawson entailment. See Smiley (1958), Tennant (1992, 1994), Ripley (2013, 2015). Ripley's program provides a particularly interesting point of connection here, as one of its key selling point is the ability to retain a constellation of plausible principles and theses in the face of paradoxes. General issues surrounding the correct formulation of transitivity are taken up in Ripley (forthcoming).

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